ABSTRACTS

Method of graphic representation of the main vector of the spatial system of forces / H. Bidnichenko, O. Belozyorova // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 5-8: Pic. 7. – Ref.: 2 title.

In many problems of mechanics are given forces acting on the solid or system of solids. Such problems are usually solved by algebraic methods. Since the force is a vector quantity, it is of interest to find a solution (to construct the main vector of the system of forces) graphically. The article presents a graphical way of representing the main vector of a spatial force system by decomposing them into two components: horizontal and frontal (vertical). The methods of descriptive geometry made it possible to represent the components of the main vector of a given system of forces in the form of projections on the Monge diagram. Two systems of vectors were obtained: horizontal and frontal (vertical), each of which was transformed into a resultant. A method for adding coplanar vectors using a power triangle is presented, with the help of which constructions are made in a horizontal system of vectors to obtain a horizontal projection of the main vector. The authors developed an algorithm for graphical addition of parallel vectors, with the help of which the frontal projection of the main vector was constructed.. The proposed algorithms are realized for the spatial system of forces from four arbitrary vectors.

Computer modelling of curves lines in natural parameterization with cubic dependence of curvature from the length of the arc / V. Borisenko // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 9-13: Pic. 3. – Ref.: 15 title.

The method of geometric modeling of parametric curves with parameter in the form of arc length, which is a natural parameter, is proposed. The curve goes through three points with known tilt angles in them. For closing the system of equations and ensuring the possibility of numerical integration, a cubic law of the distribution of curvature from the arc length, unknown coefficients, which are to be determined in the process of curve modeling, is applied. Applying the position of differential geometry, we find the dependence of the angle of inclination of the tangent to the curve on the arc length and the parametric equation of the simulated curve. **There are six unknowns in the task**. These are four coefficients of the law of distribution of the curvature and two arc lengths, from the starting point to the intermediate and from the intermediate to the final. **To reduce** the number of unknowns, the equation of the dependence of the angle of inclination of the tangent to the curve for the intermediate and endpoints is recorded, in which the angles of inclination are also known. From these dependences, expressions are used to determine the coefficients a and b. The remaining coefficients and specified arc lengths are solved by an optimization problem associated with minimizing deviations of an intermediately constructed curve (for some values of unknown coefficients and arc lengths) from given intermediate and endpoints. **Since there are** two criteria for a given task, since the curve that came out of the starting point must pass first through an intermediate point, and then through a finite point, then it belongs to the class of multicriteria tasks. In this paper, the Germeer method is used to solve it, which involves the use of a single indicator for the purpose function, in which these components are assigned different weight. **To solve** the optimization problem, a highly effective algorithm for minimizing the function of many variables was proposed, proposed by Hooke-Jeeves. **Based** on the proposed method of simulation of plane curves in natural parameterization with cubic law of the distribution of curvature, a program of calculations and visualization of the obtained results on a PC are developed. **The practical realization** of the proposed method of geometric simulation of plane curves in natural parameterization with the use of the cubic law of the distribution of curvature confirmed its efficiency.

A new approach to modelling transitional railway curves / V. Borisenko, O. Korchagina, A. Ryabova // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 14-18: Pic. 7. – Ref.: 13 title.

On the railways in the curves of the variable radius (transition curves), the radius of the track varies from infinity to the radius of a particular circular curve. The arrangement of the rail track on the curves of its sections has a number of features due to the specificity of the interaction of the track and the rolling

stock, changes in the configuration of the track in curvilinear sections and the presence of so-called transitional curves that connect the curves with adjacent straight rails or connect circular curves of different radii curvilinear The purpose of the transitional curves is to ensure the smooth change of the curvature at the points of connection of the sections of the track with different stable curvilinear rails. In this paper, the **question of geometric modeling** of the railway track, which is arranged between two rectilinear rails, located at a certain angle relative to each other, is considered. According to the traditional approach, this transition is realized as follows. Between the first rectilinear rail and the circular section, there is a transition curve, on the circular section there is another transitional curve designed for a smooth entrance to the second rectilinear rail. **The curvature** of the transition curve at the initial point is zero, and at the final point – the values of the inverse radius of the circular section. On a circular section, the curvature is a constant value. Thus, the **transition** between two rectilinear rails, located at some angle, has a circular section and two transition curves. **It should be noted** that another important feature that is accepted for consideration in the construction of transition curves is the equality of zero derivative from the curvature at the initial and final points of the transition curve. **To reduce** the number of sections in the transition between two straight lines, we simulate the transition curve of the parametric curve in its natural parameterization. Here we describe the dependence of the curvilinear on the length of the arc by the polynomial of the fifth degree. Integrating this dependence, we obtain an expression for the distribution of the angle of inclination of the tangent to the simulated curve. **Applying** the initial conditions to the law of the distribution of curvilinear and its derivative from it, we find the expressions for the five coefficients of this law. The sixth coefficient and length of the arc will be determined by solving an optimization problem associated with the coordination of the intermediately obtained end point of the transition curve with a given finite point. **Based on the** developed method of modelling of the transition curve between two straight lines of the railway way, a software code for the PC was developed, which allows to make calculations of the coordinates of the transition curves with the visualization on the screen of the obtained results.

Geometric elements that determine the nature of modeling vehicles / V. Danylenko, O. Shoman // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 19-25: Pic. 9. – Ref.: 10 title.

From the point of view of information theory, a graphic image can be considered as encoding spatial (geometric) properties of objects (images). Graphic images can be represented in different forms by the nature of the transmission of spatial forms and methods of their graphic execution. Algorithms for constructing orthogonal drawings, linear perspective and various types of central nonlinear images are used when solving a direct problem (obtaining images based on a given object) and an inverse problem (reconstruction of an object based on specified images). The specificity of relief images is that spatial objects are displayed in the space of space. It provides the opportunity to receive relief images of various types, in particular - to perform panoramic (circular) reliefs for the image of car bodies. During the visual analysis of the elements of the car body, there is a need for the use of cylindrical, conical and spherical types of reliefs. With the complication of the tasks of designing technical objects, there is a need for wider use of methods of 3D graphic modeling in various fields of production and, in particular, in the design and operation of automobile vehicles. Therefore, the development of geometric foundations for methods of 3D graphical modeling of bodywork of vehicles has a significant practical value.

Modeling of quasisurfaces on the basis of computer technologies / N. Ismailova, T. Makovkina // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 26-27: Pic. 2. – Ref.: 1 title.

The design of contact quasisurface in toothing and forming of sides of cogs of a pair of trimming gear-wheels is carried out by the concerted rotation of a blank and proper motion of an instrument. Theoretical possibilities of trimming surfaces of cog of toothing and development of contact surface of trimming quasisurface that eliminates interference were grounded. It is stated that interference will be absent in the surfaces of toothing contact if there is condition of mutual one-valued compliance. It explores the origin and expantion of quasisurface between two trimming surfaces, when millimetric and submillimetric lines appear between them and their expantion submits to the laws of geometrical optics. The design of quasisurface and its application on the base of modern computer technologies, that eliminates interference that enables to increase precision and reliability of toothing on the stage of planning has been made.

Cognitive modeling of serendipity elements Q12 based on Gaussian cubatures / E. Kremenchenko, E. Zavalko, A. Khomchenko // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 28-32: Pic. 1. – Ref.: 6 title.

The appearance of new modeling techniques and new models of serendipity elements was largely associated with physical inadequacy of standard models of Ergatoudis, Irons and Zienkiewicz. The purpose of the article is to show when enough to see a graphic portrait of zero lines to establish a priori null value of the function of random vector. An extremely informative portrait of zero lines is obtained. Zero-lines hyperboles pass through the nodes of integration of Gaussian cubature computing template and through interpolation nodes of the Q12 vector. The conclusion is obvious: in this case, we have free-load corner nodes.

Researching optimization models of distribution the pedagogical workload / K. Kuzma // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 33-40. – Ref.: 5 title.

Researching optimization models of distribution the pedagogical workload in order to increase the efficiency of this process are relevant at this time. The task of systematization of existing approaches to simulation of the distribution process of pedagogical workload is solved, the criteria of optimization of pedagogical workload distribution are determined, the analysis of mathematical methods used to describe the models is carried out. **The models** of two-criterial and multicriteria optimization are considered, for solution of which genetic algorithms are used, one-criterial model, which is described by optimization algorithms on networks and graphs, a mathematical model, which is a one-criterion task about the appointment it was established that the optimization criteria in existing models of pedagogical workload distribution are: minimization of material costs, time for the formation of annual teaching workload; uniform load of teachers of the same position with the same pedagogical workload, etc. **The above models** do not take into account the possibility of optimizing the pedagogical workload by pooling groups into streams. **As a result** of the research it was found that solving the problem of forming a rational pedagogical workload should be based on the application of information technology supporting decision-making, which will enable the head of the department or its deputies to effectively distribute types of work, taking into account restrictions and requirements.

Geometrical modeling of rotation solid state in the model of Poinsot and explanation of the Janibekov effect / L. Kutsenko, O. Semkiv, L. Zapolsky // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 41-50: Pic. 9. – Ref.: 16 title.

To study the stability of the rotation of a rigid body with a fixed point around the principal axes of the ellipsoid of inertia, it is convenient to use Poinsot's geometric interpretation. The essence of the Poinsot picture is that to determine the stability of the rotation of a body in space, it is necessary to consider the motion of its ellipsoid of inertia, which rolls without sliding along one of its tangent planes (the Poinsot plane). This plane is located perpendicular to the vector of the kinetic moment of the body and remains stationary in space. The stability of the rotation of a rigid body is determined by the shape of the rolling line of the ellipsoid, which is formed on the tangent plane and is called the herpolhode, and also by the shape and arrangement of the corresponding line on the surface of the ellipsoid – the polhode. The conclusion about the stability of this motion can be made by analyzing the location of the polhode on the ellipsoid of inertia. The classical description and definition of the geometric shape of the poles reduces to the use of elliptic integrals and is not simply formalized in the compilation of computer algorithms. From this follows the important role of graphic technologies, which make it possible to make the picture of Poinsot clearly visible, and thereby contribute to the solution of this range of tasks at a qualitative level. In this paper, the mathematical content of the computer program of the Poinsot interpretation of the rotation of a rigid body with a fixed point and the explanation on the basis of this dynamic manifestation of the Dzhanibekov effect are given.

Research of mathematical billiards in the gravitational field between two semi-planes with the use of the puankare reflection / L. Kutsenko, S. Shevchenko // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 51-62: Pic. 11. – Ref.: 10 title.

Formulation of the problem. The actual target is to compile the algorithms of mathematical billiards trajectories construction for areas specifically selected in order to identify the billiard caustics. It is expediently to develop a method of clarification the motion of the phase space point using the two-

dimensional Puankare reflection for studying the dynamics of the gravitational billiard within a certain area. **Formulation of the task**. The task is to develop a mathematical support of the algorithm of a billiard ball trajectory construction in the gravity field provided it is reflected from two rectilinear sides in the form of the angle. The next step is to give a way to clarify the motion of the phase space point using a two-dimensional Puankare reflection with the purpose of studying the dynamics of the gravitational billiard within the angle which is formed by two semi-planes. **Conclusions and prospects of the researches**. Thereby, the relations make it possible to determine the form of the set of "billiard" motion trajectories of the material point in the gravitational power field between two springy semi-planes and clarify one of the main concepts of the mathematical billiards theory which is the billiard caustic. This method of determining the Puankare reflection makes it possible to clarify the motion of the phase space point with the purpose of studying the dynamics of the gravitational billiard within the angle which is formed by two semi-planes. **Further researches** are related to obtaining the formula of Puankare reflections in case there is a hopping of the physical point to the adjacent semi-plane and both clarification and implementation Puankare reflection in practice as well.

Research of modern methods of audio content storage with losses and without loss / S. Lukyanchikov // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 62-67: Pic. 1. – Ref.: 3 title.

The paper describes formats for presenting sound data, ranging from simple time-delay representation and pulse-code modulation to complex compression formats. The results of research of universal methods of archiving of sound information with losses and losses are presented. A comparative analysis of the described methods is made. So the methods of compression of sound can be classified to ensure the integrity of sound information on: 1) lossless compression methods (FLAC, Monkey's Audio); 2) loss compression methods (ADPCM, MP3, AAC); 3) hybrid (WavPack). Also, lossy compression methods can be classified according to the principles of compression on: 1) psychoacoustic (MP3, AAC, OggVorbis, MusePack, etc.); 2) mathematical. Mathematical methods are also divided into groups: 1) wavelet methods; 2) fractal methods; 3) CDS-methods of compression. For storing music tracks, the most commonly used formats are psychoacoustic methods of compression. These methods allow fast compression in real time. High-quality audio recordings are best suited for storing either in RIFF WAVE format, or in one of the lossless compression formats that provide one hundred percent compliance with the original.

Modeling of acoustic wave process for electrical distribution in areas with central symmetry / V. Melnik // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 68-72: Pic. 3. – Ref.: 6 title.

The article is devoted to numerical modeling of wave processes in a mechanical system, including an electric discharge channel surrounding a liquid, a rigid barrier in the form of a cylindrical shell of finite dimensions, and surrounding liquid. **The article analyzes** the current state of the issue and substantiates the choice of methods for studying transient processes in liquid media and elastic shell structures; The ways of artificial limitation of computational domains in wave problems are discussed; The basic methods for calculating the emitter fields are considered. **An original approach** to the solution of spatial hydroelasticity problems is proposed, consisting in the consistent use of nonlinear equations of gas dynamics and acoustic approximation. The validity of the satisfaction of boundary conditions on a fixed surface corresponding to some intermediate position of the vapor-gas cavity is proved. This approach can be adapted to the case of an arbitrary orthogonal coordinate system. The developed procedure of calculation makes it possible to trace the evolution of wave processes in a mechanical system over long time intervals. **The algorithm is used** to solve axisymmetric and three-dimensional radiation problems by cylindrical and spherical shells, as well as their combinations. The detailed picture of the wave processes obtained makes it possible to estimate the degree of influence of the geometric features of the radiating structures. Spatial effects are revealed in the solution of non-stationary problems of hydroelasticity.

Modeling of the process of electrohydropulse treatment of melt / O. Melnik // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 73-79: Pic. 4, tabl. 1. – Ref.: 11 title.

Due to various impulse influences, the liquid metal undergoes physical processes, which leads to an increase in the quality of cast products. Thus, the study of physical processes occurring in the melt dur-

ing its processing by pulsed methods are of practical interest for foundry production. The article deals with the simulation of physical processes occurring in the melt under the influence of electrohydropulse processing, which made it possible to evaluate the effectiveness of such physical influence on the quality of cast metal. The following tasks were solved: the dynamic model of the electrodischarge oscillator generator described by the differential nonlinear second order equation is considered; A mathematical model of an axisymmetric problem on the propagation of perturbations in a melt (acoustic medium) in a two-dimensional formulation was developed; Modeling of processes of influence on the quality of castings occurring in the melt during electrohydropimpulse processing; on the basis of the obtained results the estimation of efficiency of processing regimes is given and additional possibilities of controlling the mechanism of impulse action on the processed melt for improving the quality of castings are revealed.

Constructing the equation of transformed prediction ellipse for data of the duration of the development of 3D-models of ship hull sections / S. Prykhodko, N. Prykhodko, O. Kudin, D. Koshovy // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 79-83: Pic. 2, table 1. – Ref.: 5 title.

In the bases of statistical methods of bivariate data analysis, there is the ellipse. However, well-known statistical methods (for example, outlier detection techniques) are used the assumption that the data is generated by a bivariate Gaussian distribution. But this assumption is valid in particular cases only. This leads to the need to transform the prediction ellipse for bivariate non-Gaussian data and to build an equation of transformed prediction ellipse. **In the article** for the first time the equation of transformed prediction ellipse for data of the duration of the development of 3D-models of ship hull sections is constructed. For this we have used a technique for constructing the transformed prediction ellipses on the basis of the normalizing transformations for bivariate non-Gaussian data. The technique consists of three steps. In the first step, a set of bivariate non-Gaussian data is normalized using a bijective bivariate normalizing transformation. In the second step, the prediction ellipse for the normalized data is built. In the third step, the transformed prediction ellipse for bivariate non-Gaussian data is constructed on the basis of the prediction ellipse for the normalized data and the normalizing transformation. In the article we have used SB family of the Johnson translation system for normalizing the data of duration of the development of 3D-models of ship hull sections. **An example** of applying the equation to detect the outliers in bivariate non-Gaussian data of the duration of development of 3D-models of ship hull sections is given. From the example we conclude that the proposed equation of transformed prediction ellipse for data of the duration of development of 3D-models of ship hull sections is promising.

Method of raster line dynamic smoothing / Ye. Sulema, R. Papusha // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 84-89: Pic. 8, table 1. – Ref.: 5 title.

The method of line smoothing based of the physical laws of motion of a body having its mass is proposed in the paper. The method enables dynamic updating of the smoothed line during the process of the line drawing by a user. It does not require complete redrawing of the line. The proposed method can be used in graphical editors and other applications which require interactive graphics. The results of the method application are shown and discussed. The performance of the algorithm of the proposed method is tested and compared with the algorithm of smoothing based on Bezier curves. It is shown that the performance of the proposed method is 20% higher than the performance of smoothing based on Bezier curves. The ecstatic look of lines obtained as a result of the proposed method is demonstrated as well.

Analytical presentation of buttocks of the theoretical drawing of the hull by the curves in natural parameterization / A. Ustenko, M. Gylko, I. Ustenko // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 90-94: Pic. 1. – Ref.: 11 title.

The ship's hull has a complex geometric shape. For its visual presentation in the design of the vessel is constructed theoretical drawing, which is formed by three families of curved lines: frames, waterlines and buttocks. These lines are drawn on a pre-made grid of a theoretical drawing. **Buttocks draw** after the construction of frames and waterlines. Their curvilinear character is manifested in the projection of a half-width. On two other projections, they are projected in the form of straight line segments. To facilitate the reconciliation of the theoretical drawing it is advisable to build separate buttocks, carrying out a line of communication from the frames and waterlines. **The purpose** of this work is analytical representation of

buttocks by parametric curvatures in natural parameterization with the use of linear laws of the distribution of curvature from the circumference of the circumference. The need to consider this issue is due to the fact that the theoretical drawing is 21 frames, and practical frames can be two and even three hundred. It all depends on the length of the ship's hull and the accepted spacing distance between frames. Therefore, for the determination of the geometry of practical frames, it is necessary to have analytical expressions for waterlines and buttocks. **In order** to describe the curve using the linear law of the distribution of curvature, it is necessary to know the coordinates of the initial and final points, as well as the angles of inclination in them tangents. We will submit buttocks in separate areas located between adjacent spatulas, moving from the middle of the vessel to the bow or stern finiteness. Under these circumstances, the initial and final coordinates will be known, since known applicate and abscissa of buttock's points. A slightly more complicated situation with tilt angles. At the initial portion of the buttocks, the angle of inclination has a zero value. This is due to the fact that the bottom of the vessel in its central part is a plane. **The angle** of inclination of the tangent at the end point of the initial portion of the buttocks, where the desired angle is unknown, will determine how the angle of inclination of the line, which connects the starting point of the area of the buttocks with the point of the buttocks, which is at the end of its next section.

Application of sinusoidal dependence of curvature from the length of the arc in the geometric modeling of curves lines / I. Ustenko // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 95-100: Pic. 14. – Ref.: 13 title.

The curves of the line have been widely used in solving many problems related to the geometric modeling of plane and spatial contours of various technical objects. They describe shells of ships, airplanes, cars, profiles of blades of turbines and compressors, etc. New areas of application of curves require the development of appropriate methods for constructing them. In the curve modeling, their parametric equations are often used, in which the length of the arc is taken as the parameter. **Despite** the variety of approaches to modeling the curves covered in the scientific literature, the problem of developing new methods for constructing them is still relevant, since the practice of designing technologically advanced products poses new challenges aimed at further improving the approaches to curve modeling. Thus, the problem of developing new methods for constructing curves has an important theoretical and especially practical value. In this paper, it is **proposed to simulate curves** using a modified sinusoidal law of the distribution of curvilinear $k(s) = A\sin^2(bs)+1$, where A and b are varying coefficients. In this case, the coordinates of the simulated curves were determined by numerical integration with the trapezium method of parametric equations taken in the natural parametrization. **The results** of the simulation of curves at different values of the indicated coefficients are presented in the work.

Geometrical modelling of transitionalcurves of the railway in restoration of its existing sections / S. Ustenko, V. Martynenko // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 101-105: Pic. 34. – Ref.: 13 title.

When connecting the rectilinear and circular sections of the railway, it is necessary to ensure the smoothness of the transition curve, as well as the equality of the angles of the tangent to the curvature at its ends. In this case, problems arise when the location of the rectilinear and center of the circular sections should remain unchanged, for example, when constructing a transition curve for an existing section of the railway. **In this case**, one can take a curve that is generated, provided that the graph of distribution of the curvature of the fourth order is given. Unknown coefficients of this distribution and the length of the curve of the line are determined in the process of modeling the transition curve. **Since the transition c**urve connects the rectilinear and circular sections, then at the initial point of its curvature is zero, and at the end point - the magnitude inverse of the radius of the circle of the circular section of the path. In addition, in order to avoid the appearance of a jerk angle acceleration when entering the crew into a curve, the derivative of the curvilinear in the beginning and at the end of the transition curve should be zero. **The starting data** for the geometric simulation of the transition curve will be as follows: the coordinates of the end point of the straight line section of the railway, the angle of inclination of the straight line at its final point, the radius of the circle of the circular section and the coordinates of its center. The angle of inclination of the tangent to the circular area is calculated in the process of curve simulation. With this angle you can define the coordinates of the endpoint. On the other hand, these coordinates are found using the parametric equations of the transition curve. **Since this is one and the same point**, then equating its corresponding coordinates, we obtain two equations with two unknowns relative to the first coefficient of the law of the distribution of the

curvature and the length of the arc of the transition curve. **After finding** the unknown parameters using the numerical method of minimizing the function for which the deviation of the intermediately obtained point of the end point from the given point is taken, calculate the coordinates of the transition curve, which can then be used either for the establishment of new ones, or for the control of existing rails. Thus, a **new method of geometric simulation of transition curves** is developed, which is based on the application of the distribution of the curvature of the fourth degree.

Discrete-interpolation models multiparametric environments / Yu. Kholkovsky // Geometric modeling and information technologies. – 2017. – № 2 (4), October 2017. – P. 106-108. – Ref.: 4 title.

Modeling of multiparameter systems and environments, forecasting their condition is a rather complex and at the same time an extremely important practical task, as well as a socio-social problem that arises in the process of processing the results of monitoring such environments and elaborates certain recommendations and methods for long-term forecasting of their condition, determination of anthropogenic impact. In the conditions of uncontrolled human impact on the environment, the current global environmental crisis work on the organization of environmental monitoring, processing its results and forecasting the state of a particular ecosystem becomes of particular importance. By all indications, multi-parametric environments such as ecological, energy, hydrological, meteorological, etc. can be categorized as stochastic systems. This follows from the fact that such systems and environments are often interconnected and it is impossible to view a separate system in isolation. It is obvious that for such systems and environments it is impossible to create their continual model. In our opinion, it may be appropriate to use discrete mathematical models, namely geometric, in the form of discrete numerical arrays, elements of which are certain components of systems and environments. The discrete-interpolation method of multi-parametric objects, systems and environments simulation is based on the use of certain discrete-interpolation schemes using Lagrange interpolation polynomials. On the basis of certain interpolation schemes one-parameter sets are obtained which are discrete mathematical models of certain processes, systems and environments.